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AGRICULTURAL MEMORANDUM NO.916

An assessment of the accuracy of accumulated degree-days calculated from hourly, daily and monthly temperatures above various bases

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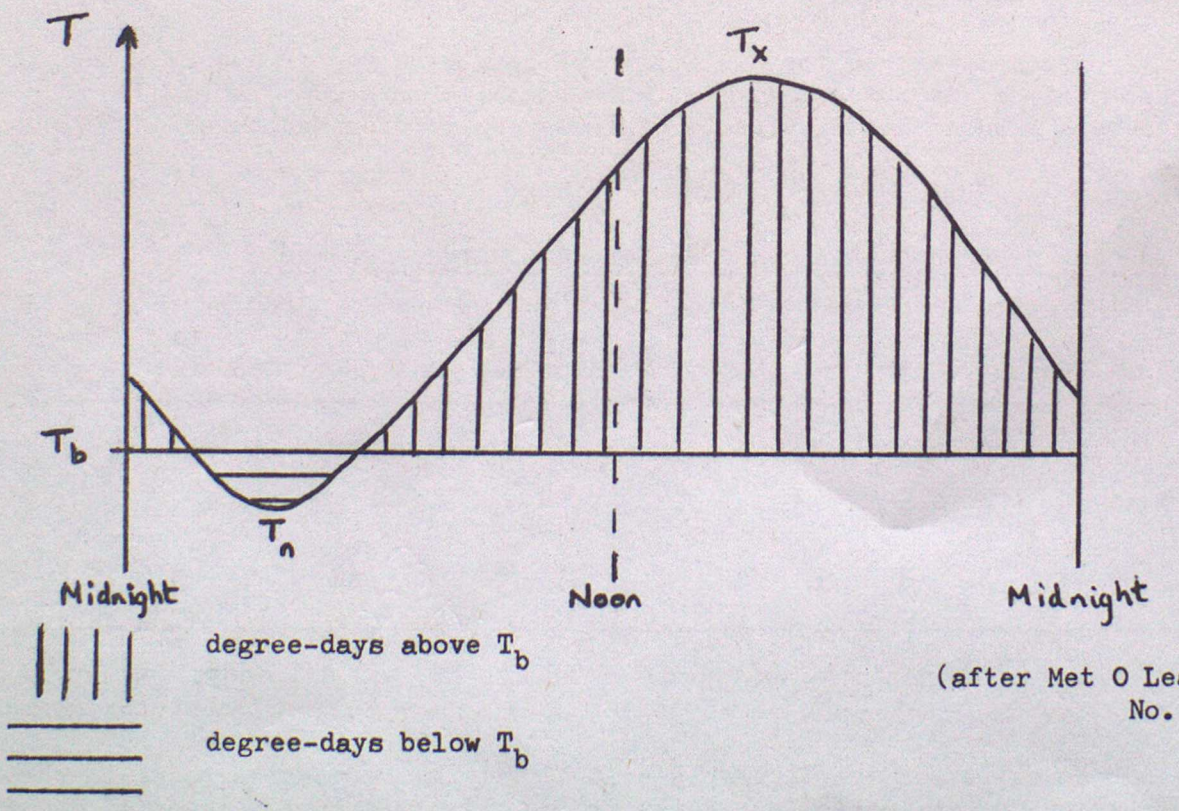
Accumulated degree-days (dd) have often been used in studies relating plant growth to air temperature. Degree-days above 10°C calculated on a daily basis are used for assessing the progress between phenological stages of crops such as maize (1), while degree-day summations below various thresholds are often used to estimate the glasshouse heating losses (with due consideration to wind regime a necessary input).

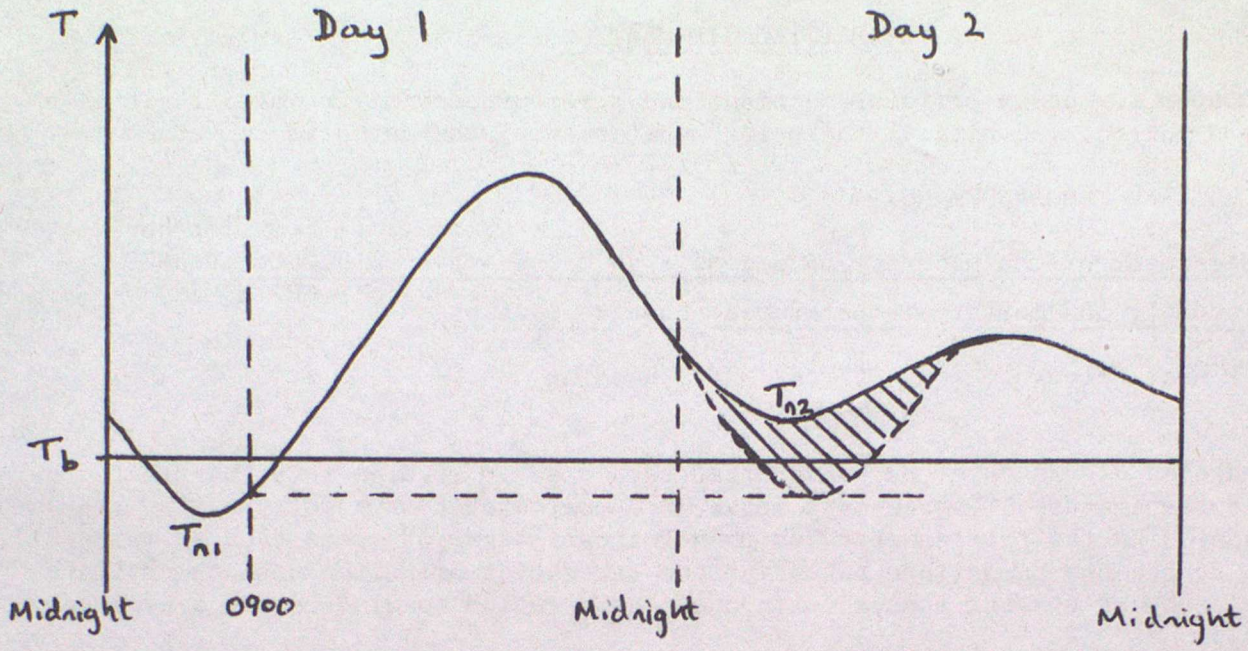
The method of evaluating daily degree-days above and below 42°F (5.6°C) from daily maximum and minimum temperatures was described in detail in Met. O. Leaflet No.10 (2). Essentially this involves fitting a sinusoidal wave form through the daily max. (T_x) and min.(T_n), the area bounded by the base temperature (T_b) and the temperature curve representing the number of 'daily dd' contributions (see diagram)



(after Met O Leaflet No. 10)

Daily max. and min. temperatures used in the calculations are recorded over the period 0900 - 0900 GMT; by convention the maximum reading at 0900 is 'thrown-back' to the previous afternoon while the minimum is assumed to have occurred in the early hours of the morning on which the reading was taken. The problem therefore arises when the maximum or minimum, or both, do not occur at the expected times. Such a

situation is shown in the following diagram.



Consider a change for example from anticyclonic conditions with cold radiation nights (min.T_{n1} on Day 1) to mild cyclonic weather during day 1 after the passage of a warm front. It is likely that the min.on day 2 (T_{n2}) will exceed the 0900 dry-bulb temperature for the previous day. Thus the min. for day 2 will equal the 0900 air temperature on day 1. On fitting a sinusoidal curve, an error equivalent to that shaded in the diagram will result.

A data sorting program was run for 3 stations to determine the frequency with which the day-minimum was lower than the following night minimum (Table 1). Temperatures were grouped into intervals of 2 degrees to ease manipulation.

TABLE 1

Percentage occasions (rounded up) when day minimum < night minimum
1970-79

Station	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	May Oct	Nov-Apr
Gatwick	24	16	10	4	1	1	2	0	2	10	17	25	3	16
Hurn	20	19	7	3	2	2	1	1	3	10	22	19	3	15
Manston	23	15	8	3	2	2	1	2	2	10	18	24	3	15

From Table 1, this phenomenon occurred on about 3% of all days on average during May to October and on about 15% of all days during the winter months.

Interest has been expressed, particularly by ADAS Viticulture advisers, as to the validity of degree-day estimates from daily and monthly data in the light of anomalies such as these and this paper attempts to put into context the magnitude of the errors involved.

Method

Hourly summations (using the Trapezium rule) of mean hourly temperatures (T_m) above 0°C, 6°C and 10°C were made for 0900 - 0900 GMT each day (to correspond with estimates using daily maximum and minimum temperatures) and accumulated over the month. Be meaning consecutive dry-bulb temperatures it was hoped to reduce 'end-errors' when two observations straddled the base. Therefore:

$$\text{hourly dd above } T_b = \sum_{i=i_1}^{i_2} \frac{(T_{mi} - T_b)}{24}$$

where (i_2 = mean for hours 0800 and 0900 GMT on day 2

i_1 = mean for 0900 and 1000 GMT on day 1)

$$\text{hourly dd below } T_b = \sum_{i=i_1}^{i_2} \frac{(T_b - T_{mi})}{24}$$

Data for Gatwick, Hurn and Manston were calculated for each month, 1971 - 78 and compared with the corresponding 'daily' estimates. Figures for Ringway have been included to see if the differences become more pronounced in a cooler environment. Actual and percentage differences between estimates are presented where:

$$\% \text{ difference} = \frac{\text{daily dd} - \text{hourly dd}}{\text{hourly dd}}$$

Positive values therefore indicate an over-estimate by using daily maximum and minimum.

Monthly mean temperatures have been used in various forms to estimate monthly and seasonal degree-day totals. Several methods were considered above a base of 10°C at Gatwick for 1971 - 78 and compared to the hourly summations. The methods used were briefly as follows:

1. In Agricultural Memorandum No. 527 (3) Harvey describes a method whereby Thom's formulae for estimating degree-days above or below any base are presented in graphical form, thus eliminating the need for calculation. For a 28, 30 and 31 day month a family of curves (Figs 1, 2 and 3) were produced which could be used to estimate degree-day totals for the month above (or below) any base. For this two parameters are required:

   i) the difference between the monthly mean temperature and the base, and

   ii) the standard deviation of the monthly mean temperature. The method is particularly useful in that degree-day contributions for the month above the base are not counted as zero when the monthly mean temperature falls below the base.

2. Bunting (4) derived linear regression equations for estimating monthly degree-days above 10°C (y) from monthly mean temperatures °C (x) using data from 9 sites in the Central Lowlands of England. These equations are given in Table 2.

Table 2

Joint regression equations ($y = bx - a$) for estimating monthly degree-day totals above 10°C (y) from monthly mean temperature °C (x)

	May	June	July	Aug	Sept	Oct
a.	169.8	232.2	279.0	276.1	220.8	117.1
b.	21.80	26.27	29.42	29.28	25.38	16.92

3. Seasonal totals of degree-days above 10°C may be predicted from (4):

$y = 23.8 x - 1180$ (inland areas)

and $y = 25.3 x - 1335$ (coastal areas)

where y is the season's total.

Results

1. Using daily maximum and minimum temperatures to estimate monthly degree-day totals.

The average differences, plus extremes, between hourly and daily estimates of monthly degree-day totals above 0, 6 and 10°C are shown in Figs. 4, 5 and 6 respectively. The figures show that the daily estimate might be in error by as much as $\pm$20 degree-days in any one month (where positive values indicate that the daily method over-estimated the hourly total and vice-versa). The discrepancies for individual months, over the period 1971-78, was less than $\pm$12 degree-days on average.

Above 10°C, the daily method generally over-estimated the hourly value (particularly at Manston) while above 0°C the proportion of over-estimates was approximately equal to the number of under-estimates. (See Table 3)

Table 3

Percentage of occasions when monthly totals from daily calculations exceeded the corresponding hourly value for 1971-78 at Gatwick, Hurn Manston, and Ringway combined.

Base	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
0°C	9	25	81	94	63	41	81	88	78	63	34	6	55
6°C	38	88	97	97	69	44	78	91	91	81	44	28	70
10°C	100	100	100	100	97	69	91	94	97	94	72	81	91

The pattern of errors at all four stations was similar although Manston recorded the largest differences while Ringway consistently produced the smallest errors.

2. Comparison of methods to estimate 6 monthly totals

A comparison of hourly and daily summations for May - October and November - April at Gatwick, Hurn and Manston combined is shown in Fig. 7. It is

apparent that the summer values are inherently more reliable, probably due to the diurnal temperature variations following a more conventional timing pattern. Above 10°C, for instance, less than 1 in 10 of the discrepancies indicated an over-estimate by the daily values of greater than 10% during May - October, while for November - April over 90% of the errors calculated were greater than 10%. Above bases of 0 and 6°C, the errors for May - October lay entirely within ±5% of the hourly value, and totally within ±20% for November - April figures. Again the magnitude of the errors increased as the base temperature was increased.

The 1971-78 average percentage errors for May - October and November - April for Gatwick, Hurn, Manston and Ringway are shown in Table 4.

Table 4

The average seasonal % differences* between hourly and daily degree-day calculations at Gatwick, Hurn, Manston and Ringway, 1971-78

Station	Above 0°C		Above 6°C		Above 10°C	
	May-Oct	Nov-Apr	May-Oct	Nov-Apr	May-Oct	Nov-Apr
Gatwick	0	-1.5	1.6	2.9	4.6	26.0
Hurn	0.3	-2.5	0.2	-0.4	2.8	27.8
Manston	2.0	1.1	3.8	10.9	8.3	46.1
Ringway	0.3	-1.1	0.9	2.4	3.4	23.3

* positive values indicate that daily value > hourly value

Thus the results between stations are fairly consistent apart from Manston where relatively large overestimates by the daily method were recorded (although the daily value did under-estimate the hourly value in some years during January, February, November and December).

The average, plus extreme, May - October percentage differences between seasonal totals estimated by Thom's formulae, Bunting's and T.B.35's regression formulae and the hourly values are shown in Table 5 for Gatwick above 10°C.

Table 5

Percentage errors in using various methods to estimate May-October degree-days above 10°C at Gatwick, 1971 - 78.

Method	Average 1971-78 % error	Range
Thom	-3.7	-6.6 - 1.9
Bunting	4.1	-0.4 - 6.3
Tech Bul 35	0.1	-5.2 - 4.7
Daily max and min	4.6	-0.6 - 7.3

Thus the Tech.Bul. 35 method achieved the best estimates from the small sample considered. The anomaly is any single year could be expected to be less than 10% for most stations and of the order of 5% using average values.

Conclusions

If it is assumed that a tolerance of within $\pm$5% is an acceptable error between estimated values and actual values, then the use of daily max and min to predict monthly totals (May - October) are unlikely to meet these tolerances on approximately half of the occasions considered above a base of 10°C. At higher bases this uncertainty is likely to increase. For bases less than 10°C, the daily method may be used with some degree of confidence that the tolerances will be met. A similar pattern also exists for May - October totals using the daily max.and min. method, with errors in any one year exceeding $\pm$5% on 50% of occasions above T_b = 10°C; for November - April the differences were considerably higher and the use of daily temperatures is not recommended except for accumulation above low base temperatures. For providing estimates of the average number of degree-days in any month, or group of months, the daily method is more satisfactory, and generally would be less than 5% in error. Even so, the average monthly value might be in error by over 10% during May or October above a base of 10°C.

The use of Thom's, Bunting's and Tech. Bul 35 methods to predict the average May - October totals above 10°C appears successful, but again for individual years the error might exceed $\pm$5%.

References

1. MAFF Technical Bulletin No. 35 (1976). The Agricultural Climate of England and Wales. HMSO London.

2. Met.O. Leaflet No.10. Tables for the evaluation of daily values of accumulated temperature above and below 42°F from daily values of maximum and minimum temperature. Met.Office, London Road, Bracknell, Berks.

3. Harvey, L D J (1973). Simple computation of degree-days above or below any base. Agricultural Memorandum No. 527.

4. Bunting, E S (1979). The relationship between mean temperature and accumulated temperature in the central lowlands of England. J. Agric. Sci. 93, 157-169.

Fig 1

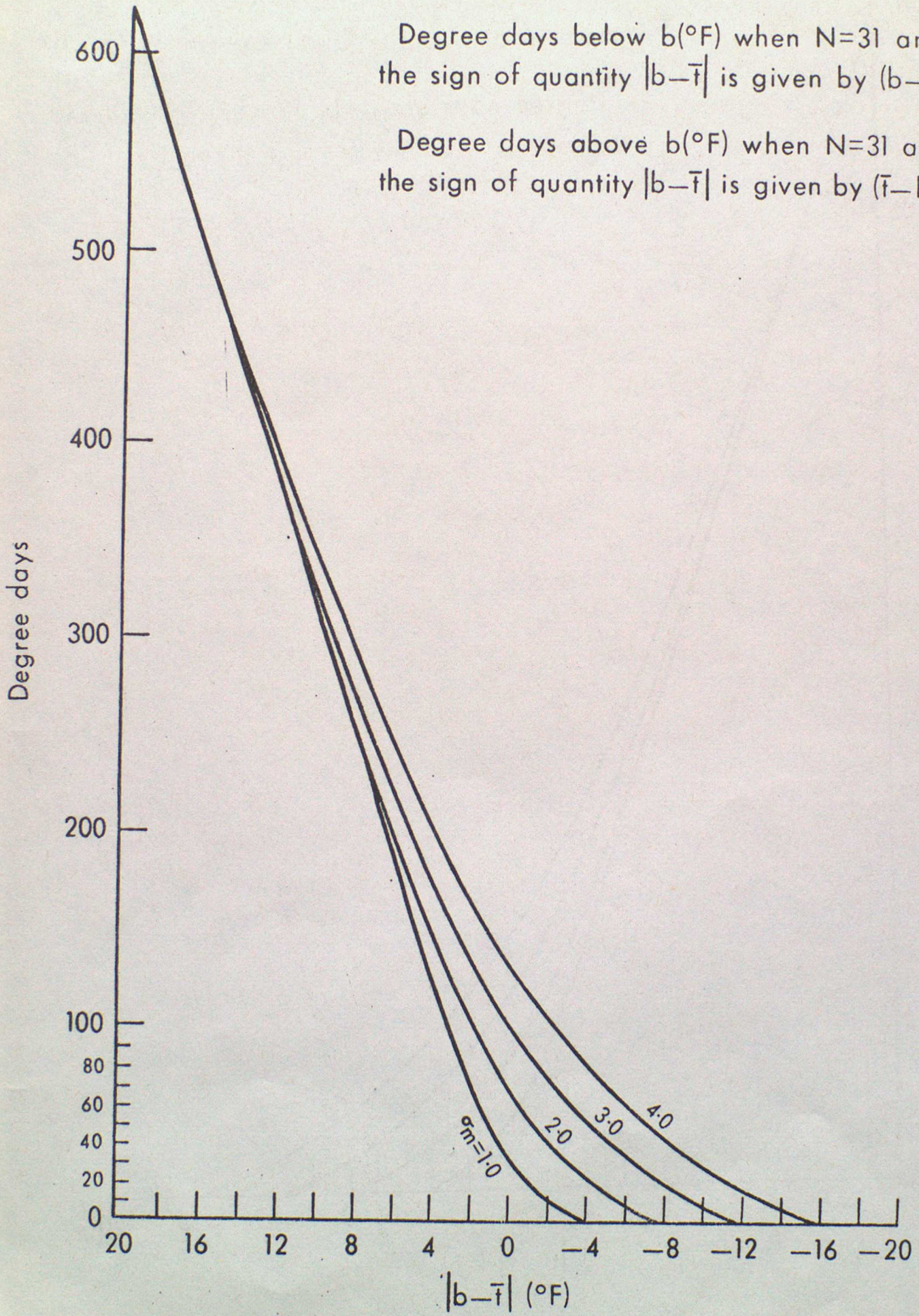

Degree days below b(°F) when N=31 and the sign of quantity |b−t̄| is given by (b−t̄)
Degree days above b(°F) when N=31 and the sign of quantity |b−t̄| is given by (t̄−b)
Degree days
600
500
400
300
200
100
80
60
40
20
0
σm=1·0
2·0
3·0
4·0
20
16
12
8
4
0
−4
−8
−12
−16
−20
|b−t̄| (°F)

Fig. 2

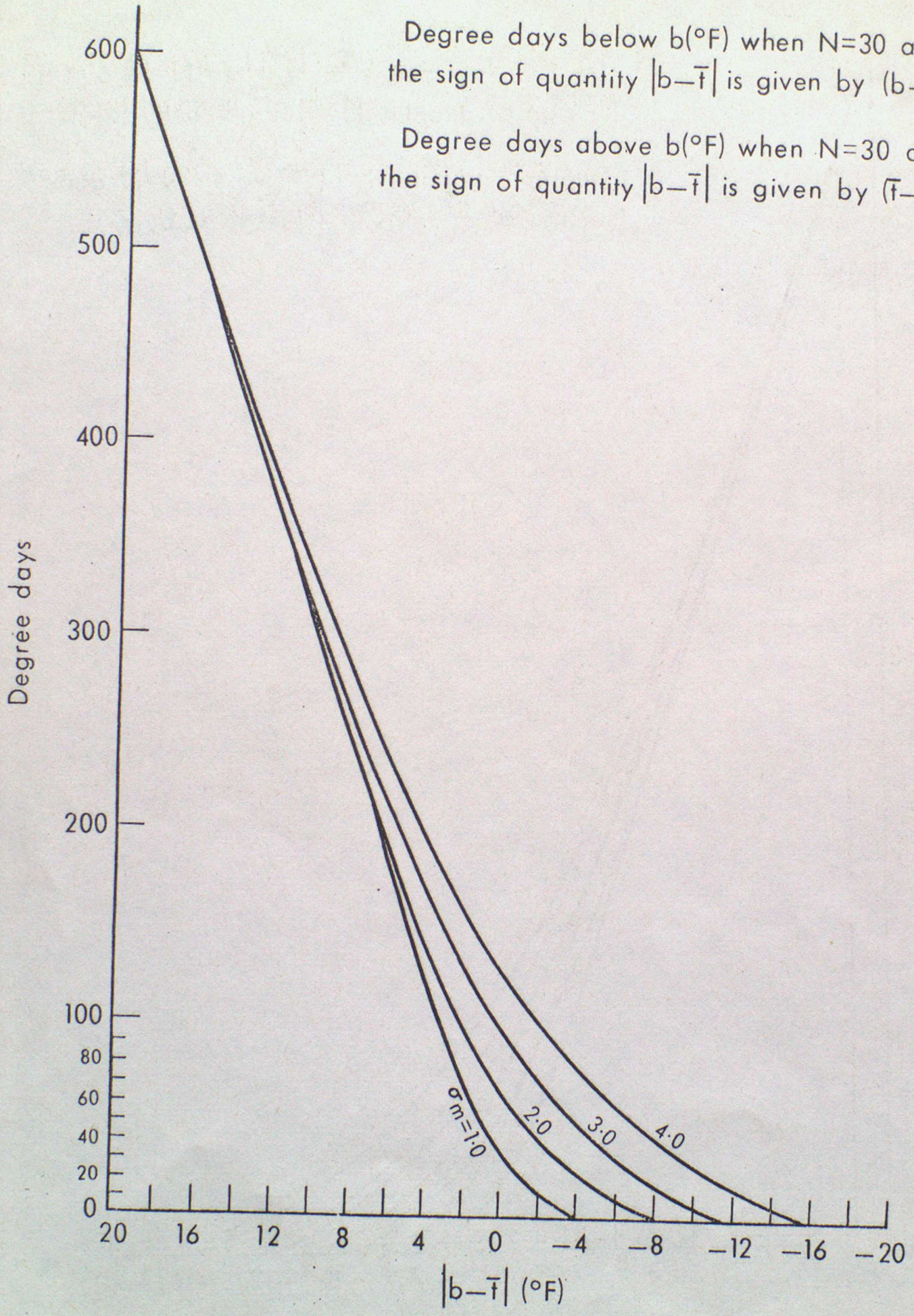

Degree days below b(°F) when N=30 and the sign of quantity |b−t̄| is given by (b−t̄)
Degree days above b(°F) when N=30 and the sign of quantity |b−t̄| is given by (t̄−b)
Degree days
600
500
400
300
200
100
80
60
40
20
0
σm=1·0
2·0
3·0
4·0
20
16
12
8
4
0
−4
−8
−12
−16
−20
|b−t̄| (°F)

Fig 3

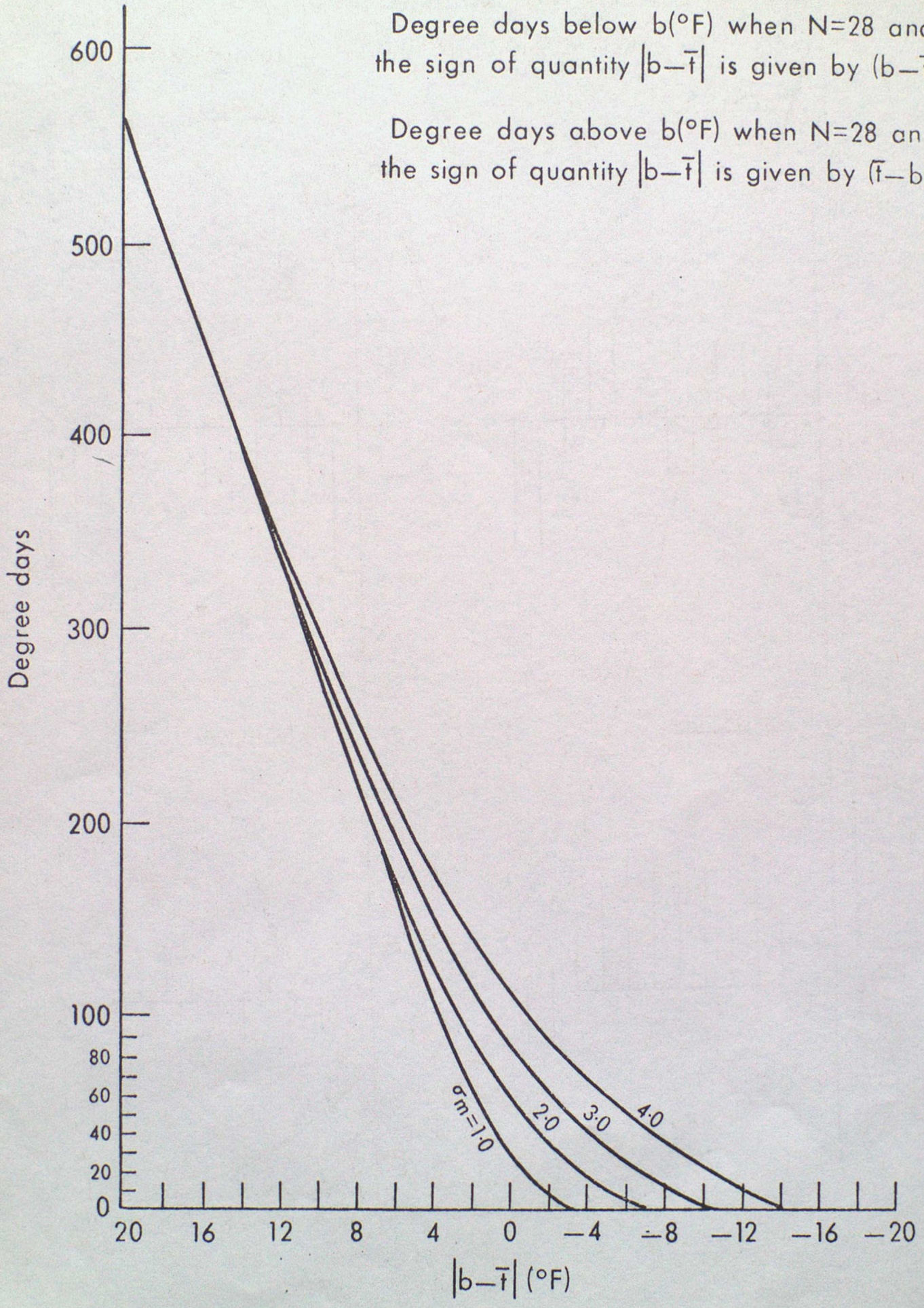
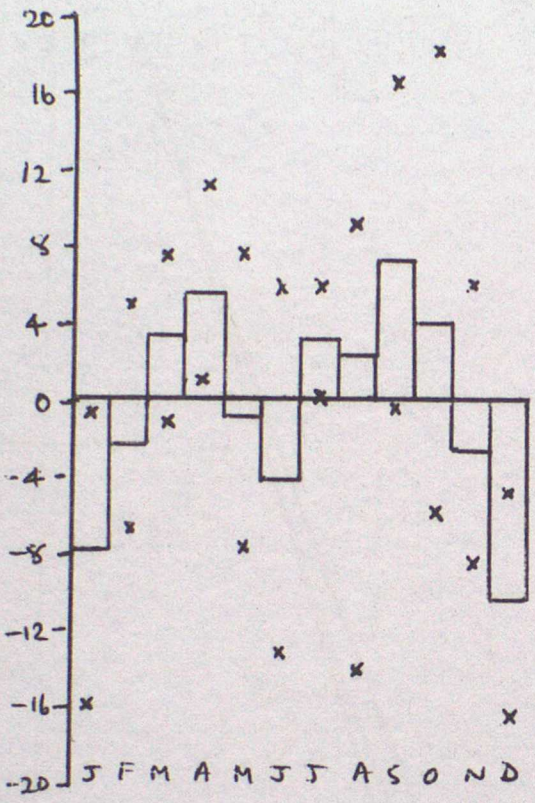

Degree days below b(°F) when N=28 and the sign of quantity |b−t̄| is given by (b−t̄)
Degree days above b(°F) when N=28 and the sign of quantity |b−t̄| is given by (t̄−b)
Degree days
600
500
400
300
200
100
80
60
40
20
0
σm=1·0
2·0
3·0
4·0
20
16
12
8
4
0
−4
−8
−12
−16
−20
|b−t̄| (°F)

Fig 4

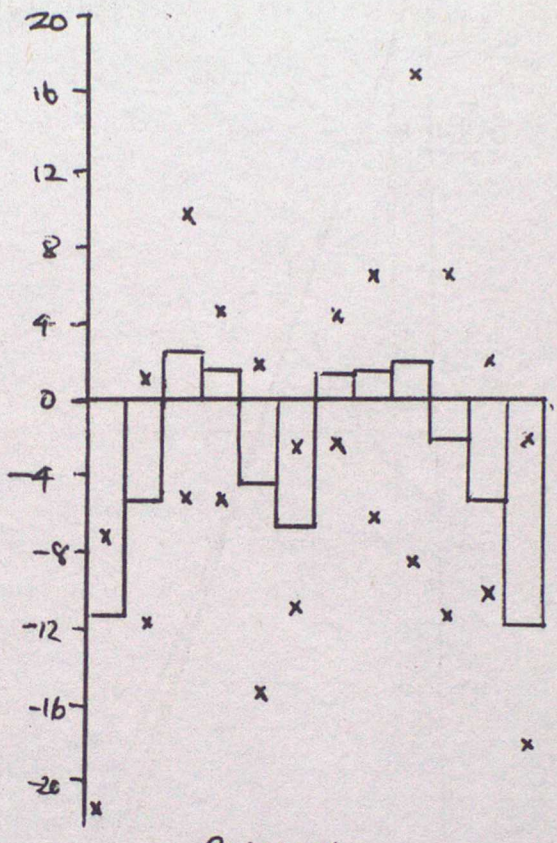
AVERAGE DIFFERENCES, PLUS EXTREMES, BETWEEN HOURLY AND DAILY DEGREE-DAY ESTIMATES ABOVE 0°C, 1971-78

+VE VALUES INDICATE DAILY > HOURLY ESTIMATE

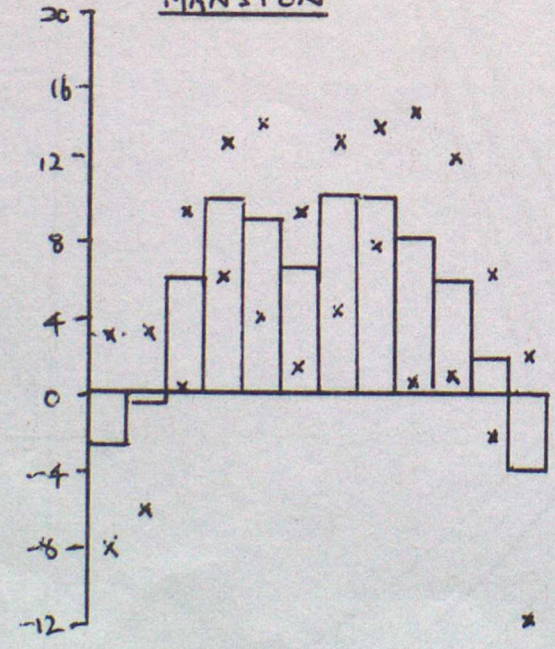
GATWICK



HURN



MANSTON



RINGWAY

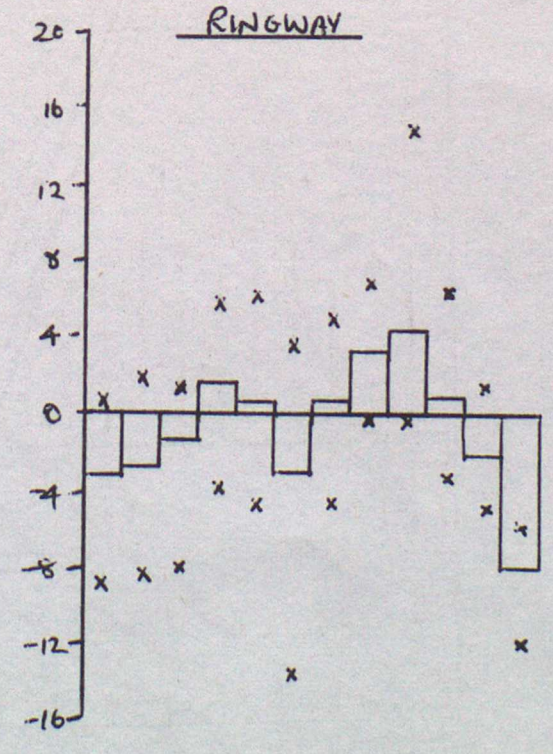
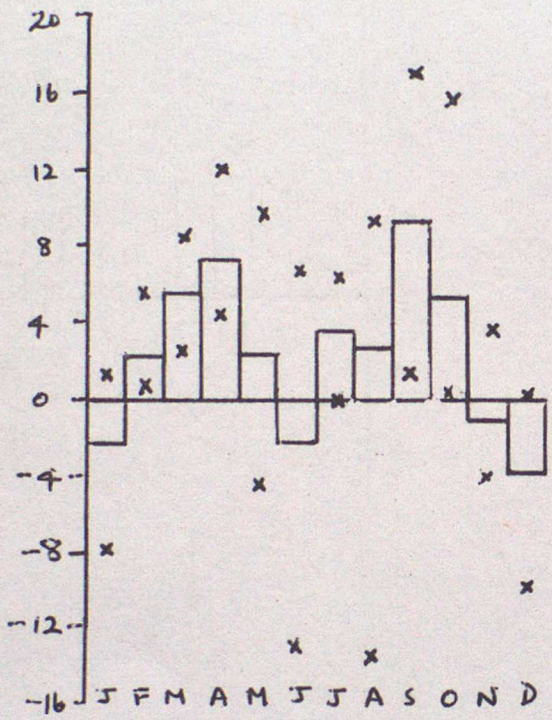


Fig. 5.

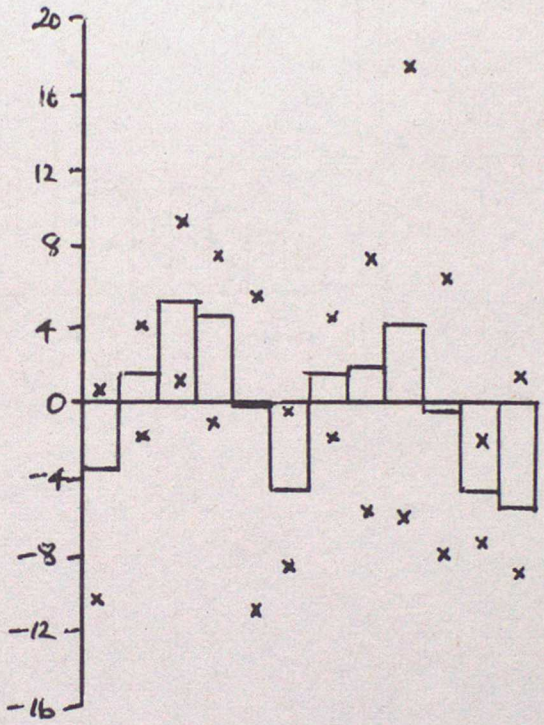
AVERAGE DIFFERENCES, PLUS EXTREMES, BETWEEN HOURLY AND DAILY DEGREE-DAY ESTIMATES ABOVE 6°C, 1971-78

+VE VALUES INDICATE DAILY > HOURLY ESTIMATE

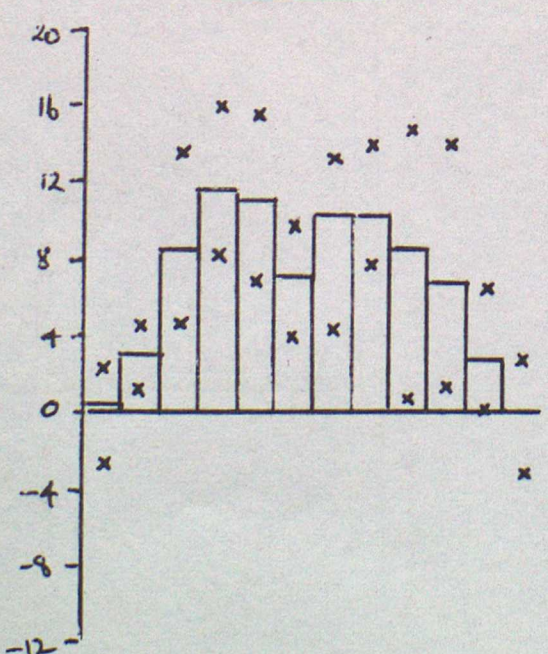
GATWICK



HURN



MANSTON



RINGWAY

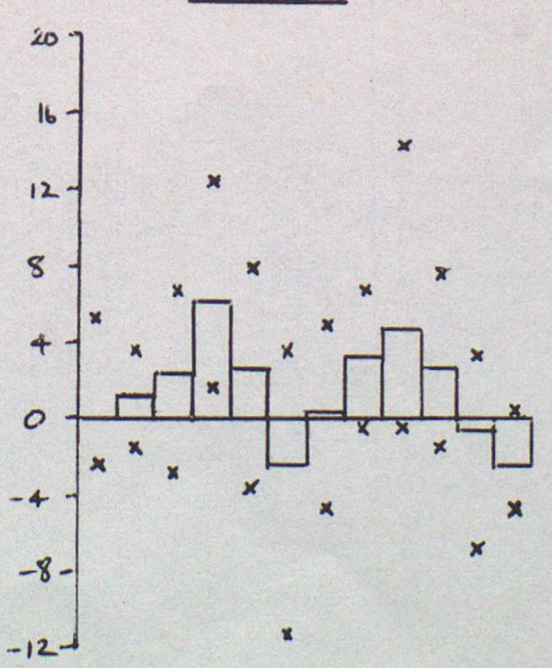


Fig: 6.

AVERAGE DIFFERENCES, PLUS EXTREMES, BETWEEN HOURLY AND DAILY DEGREE-DAY ESTIMATES ABOVE 10°C., 1971-78

+VE VALUES INDICATE DAILY ESTIMATE > HOURLY ESTIMATE.

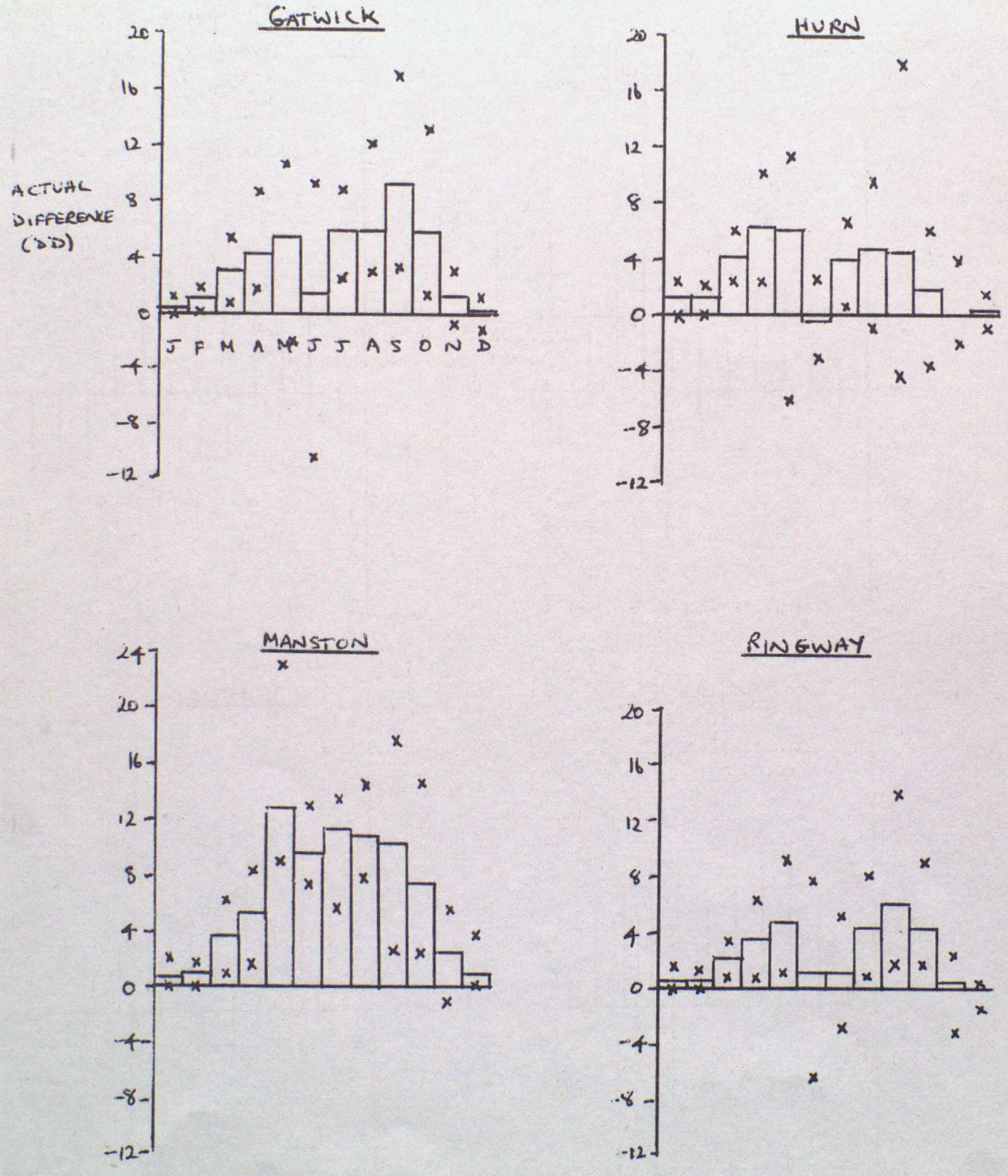


Fig 7.

ANALYSIS OF 6-MONTHLY % ERRORS BETWEEN HOURLY AND DAILY DEGREE-DAY CALCULATIONS FOR MAY-OCT AND NOV-APR AT GATWICK, HURN AND MANSTON COMBINED (1971-78/79)

KEY

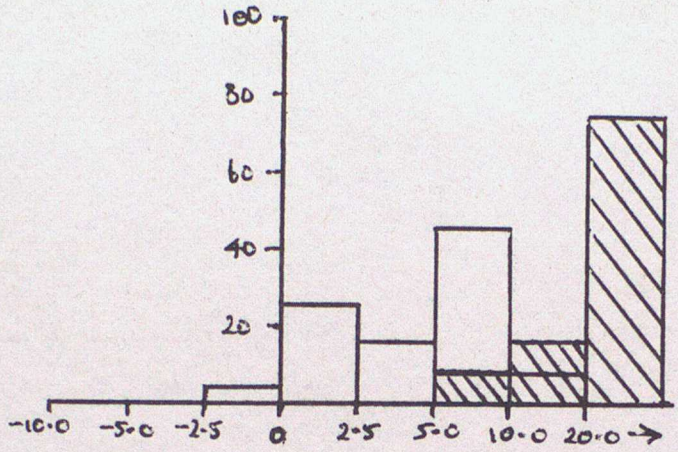
MAY - OCT

NOV - APR

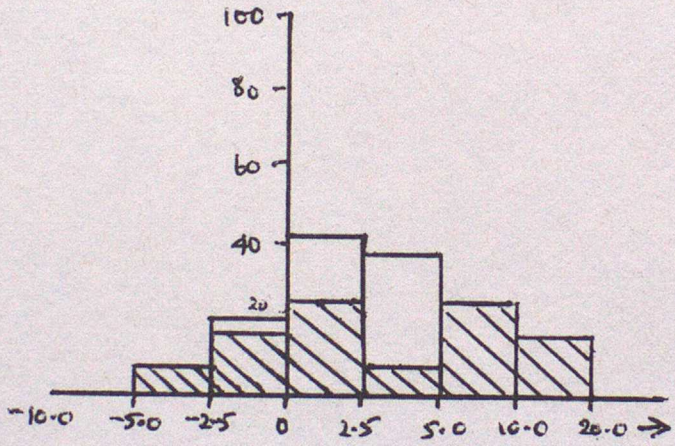
VERT. AXIS: % OF MAX POSSIBLE

HORIZ. AXIS: % ERROR (+VE IF DAILY > HOURLY VALUE)

ABOVE 10°C



ABOVE 6°C



ABOVE 0°C